Frtool – The User's Guide

Frequency Response Controller Design Tool

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1.1. Introduction

Control engineers/students can design a controller in one of following ways: applying a mathematical design method or using a controller design tool. In case they choose to apply a mathematical method, they usually have to simplify the mathematics. Using a controller design tool, they can tune the controller's parameters without simplifying the process (transfer function) and can obtain, in most of the cases, better (best) closed-loop performance.

At this time, Matlab offers a controller design tool based on the root locus method (rltool). This tool cannot handle systems with time-delay without approximating the dead-time by a rational transfer function. To solve this problem, we choose to work with frequency diagrams (Nichols diagrams), where we know that dead-time (τ_d) will have no influence on the gain but it will just shift the phase with ($-\tau_d*\omega$).

1.2. Design specs

An important feature of a controller design tool is the possibility to define design constraints, which will guide the designer in the tuning process. These design specs have to be converted into graphical restrictions to make designer's job easier. Graphical restrictions are obtained from numerical specs using relations defined for the second order system, relations which can be extended to higher order systems as a rule of thumb.

We considered as important design specs the following ones: robustness, settling time, maximum overshoot, gain and phase margins .

Given the maximum overshoot as design specification we can compute the damping factor (ξ) using relation (1.1):

$$OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad (1.1)$$

Straightforward we can obtain the peak value (M_r) of the closed-loop frequency-domain magnitude (Bode) as defined in relation (1.2), which will be a curve on the Nichols plot as shown in Figure 1.1. You can see that increasing the value of OS from 5% to 35% the curves become less restrictive (they become smaller) .

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad (1.2)$$

$$t_s \cong \frac{2\zeta^2 + 3}{\zeta\omega_n} \qquad (1.3)$$

To fulfil this specification, the Nichols curve of the open loop system has to be outside or tangent to the correspondent M_r curve.

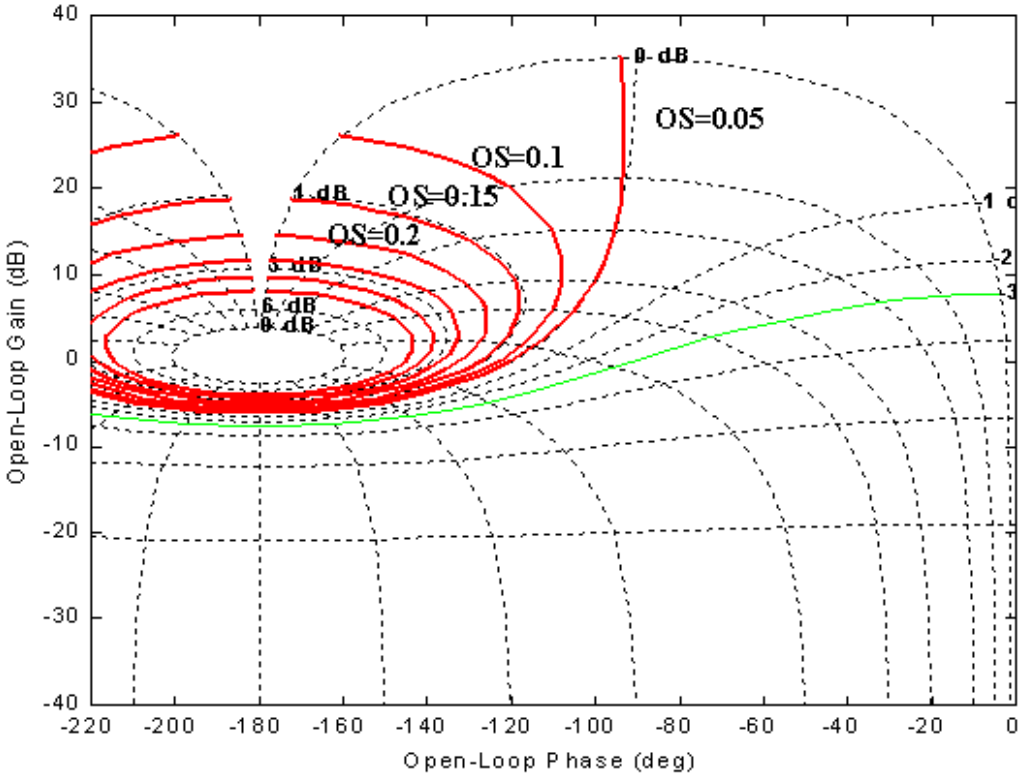

Figure 1.1. Constant M_r curves corresponding to overshoot values

The second important design specification is the settling-time. Given the settling-time and the maximum overshoot specs we can use relations (1.1) and (1.3) to obtain the natural frequency (ω_n) of the equivalent second order system and the damping factor. Once we know the damping factor and the natural frequency, the value of the –3dB bandwidth frequency for the closed-loop is obtained with relation (1.4). It will be used as

graphical restriction because frequency is the parameter along the Nichols curve.

$$\omega_b = \omega_n \sqrt{1-2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \qquad (1.4)$$

Figure 1.2 shows how the bandwidth frequency is displayed on the Nichols plot.

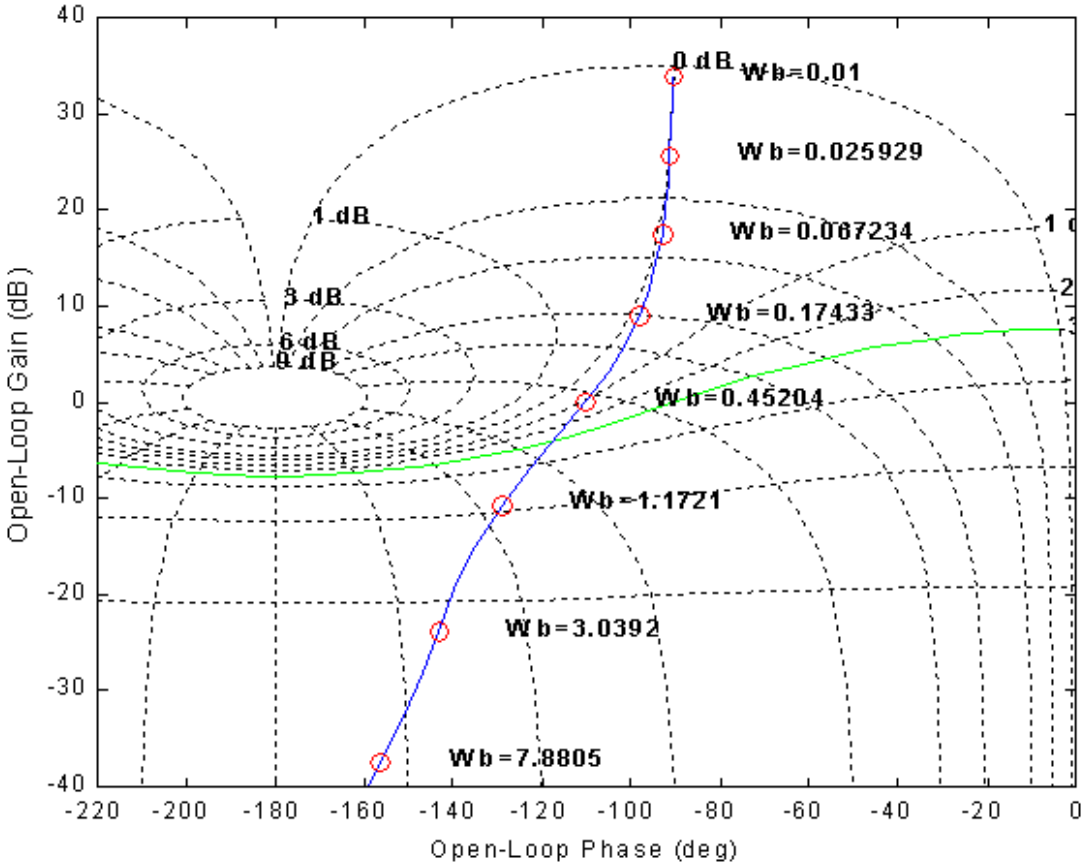

Figure 1.2. Nichols plot with highlighted frequencies

You can see that along the Nichols curve there are multiple red circles corresponding to the variation of (ω_b). To fulfill the settling-time design specification, the circle corresponding to the related value of (ω_b) has to be above the green –3dB line and, because the value of (ω_b) was obtained using also the overshoot spec, the overshoot spec has to be respected too.

Robustness design spec can be taken into account according to the following definition in the Nyquist plot (see Figure 1.3). Robustness is the radius of the circle and it can have values that respect the relation 0<Ro<1. Because we are using the Nichols plot for the design process, we need to convert this robustness spec from Nyquist to Nichols. Given the value of (Ro) we can compute the values of the real and imaginary part of the Ro vector with relation (1.5) and (1.6), where ω is the angle of Ro vector measured as shown in Figure 1.3.

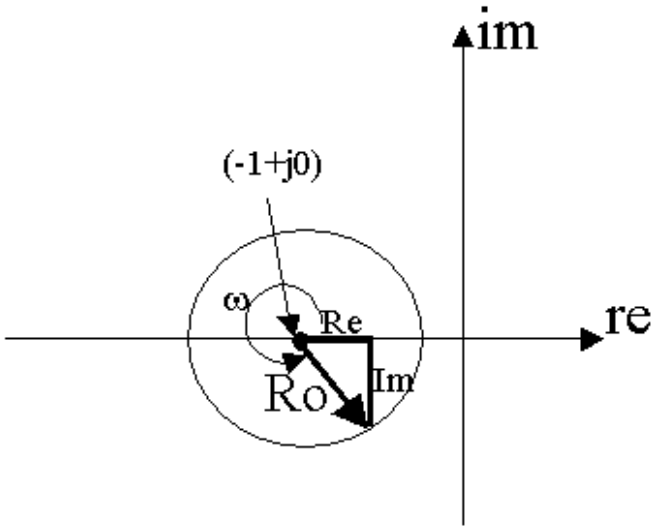

Figure 1.3. Definition of robustness on Nyquist

$$\mathrm{Re} = -1 + \mathrm{Ro} \bullet \cos(\omega) \qquad (1.5)$$

$$\mathrm{Im} = \mathrm{Ro} \bullet \sin(\omega) \qquad (1.6)$$

$$\mathrm{M_{dB}} = 20 \bullet \log 10\left(\sqrt{\mathrm{Re}^2 + \mathrm{Im}^2}\right) \qquad (1.7)$$

$$\phi_{\mathrm{degr}} = -180 + \arctan\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right) \bullet \frac{180}{\pi} \qquad (1.8)$$

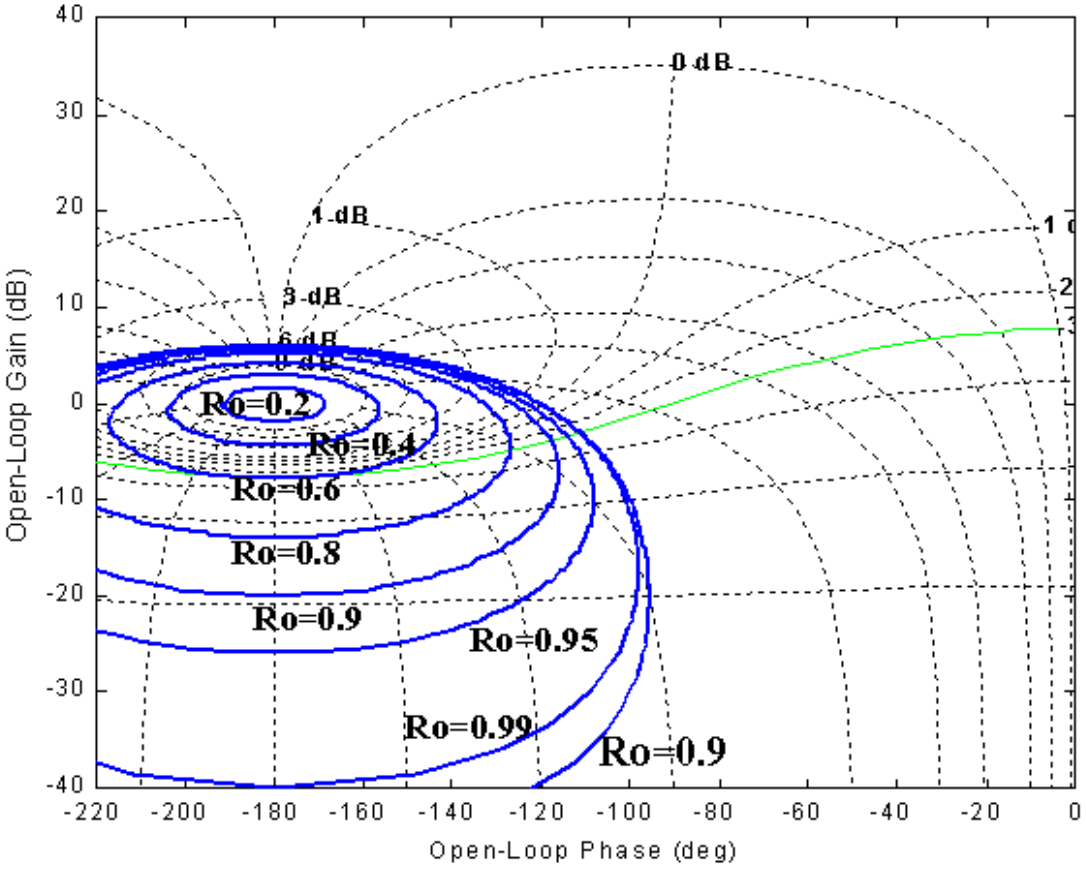

Figure 1.4. Robustness curves on Nichols plot

Once the real and imaginary parts of a complex number are known, its modulus and phase can be computed using relations (1.7) and (1.8). As a result of this conversion on the Nichols plot we will obtain the curves shown in Figure 1.4 corresponding to the circles from Nyquist .
Robustness spec is fulfilled only when the Nichols curve of the open-loop system remains outside the restriction curve .

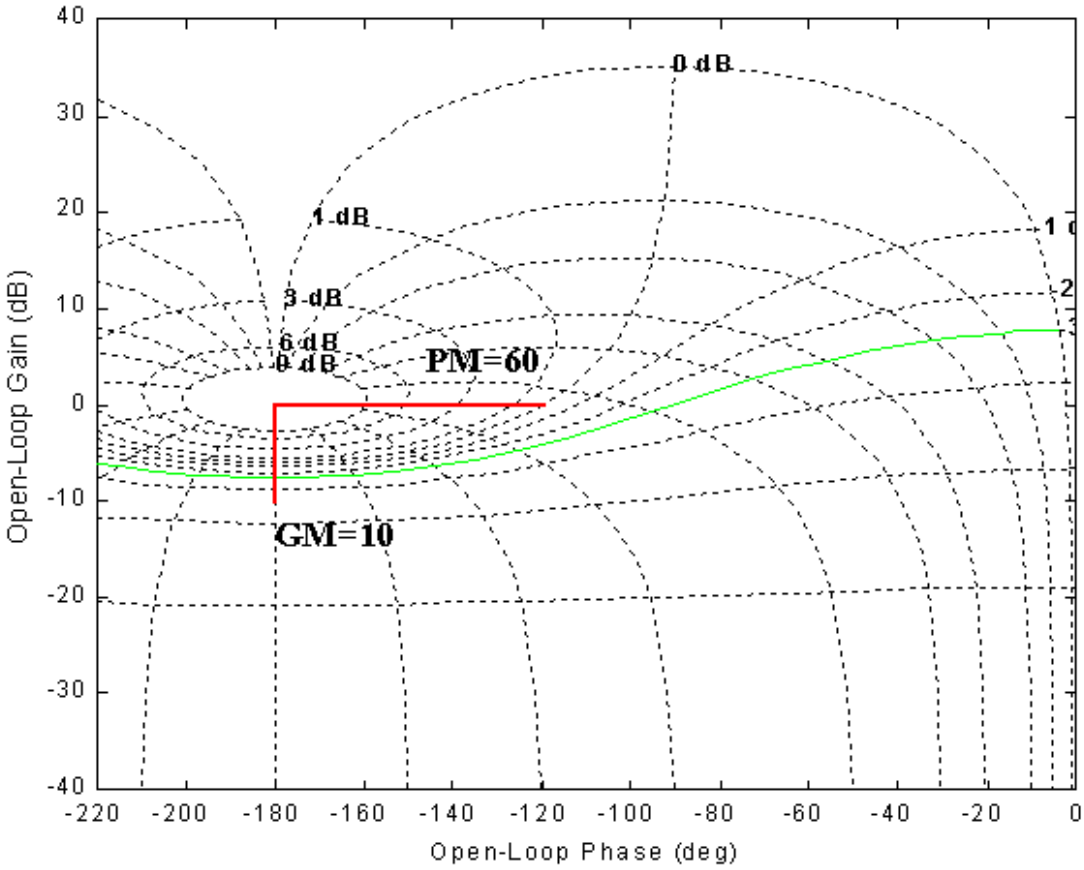

Figure 1.5. Gain and phase margins measured on the Nichols plot

Besides these 3 restrictions we also considered gain and phase margins, because these two design specs are easy to read on the Nichols characteristics. The gain margin is the vertical line that starts in the point having 0 dB gain and –180 degrees phase and goes downward, while the phase margin restriction is the horizontal line starting in the same point and going to the right side of the graphic. Figure 1.5 illustrates these explanations. To fulfill these two specs the Nichols curve has to go on the right side of the phase margin line and below the gain margin line, without crossing any of them .
With all design specs active and fulfilled the Nichols plot from the main window of the tool should look like the one shown in Figure 1.6 .

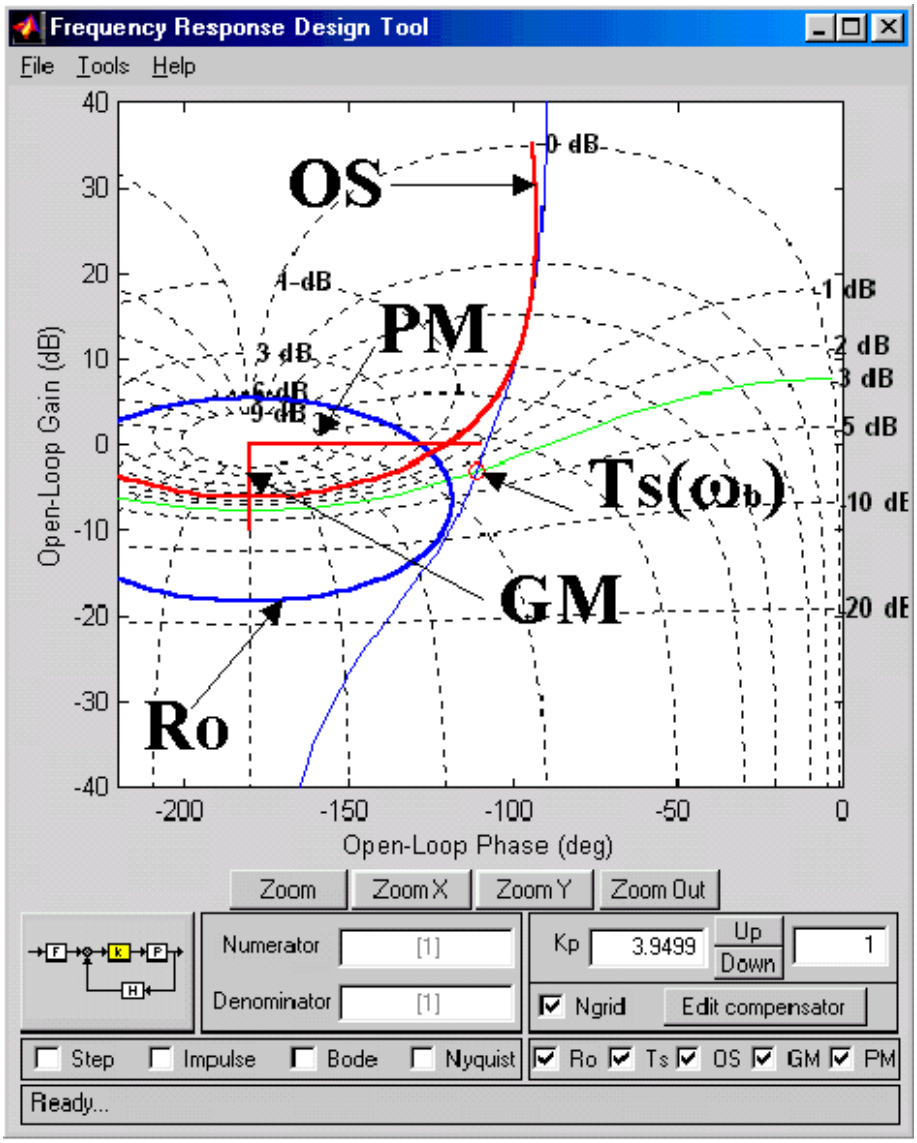

Figure 1.6 Main window of FRTool
(Specs fulfilled on Nichols)

1.3. GUI

The FRtool's graphical user interface (GUI) is powerful, offering a set of useful features that make this tool easy to use. It has the following windows:

- one in which the Nichols curve of current open loop and restrictions are shown and which is used to set design specs, do file operations and design a part of the controller (its gain); from now on we will call it "**main window**";
- one in which the user can see step and impulse responses as well as Bode or Nyquist characteristics, used for checking whether design specs are fulfilled or not; this one will be called "**responses window**";
- one for placing controller poles and zeros; this will be called "**controller design window**" because this is the window where the user will "play" with controller's parameters;

- one in which the user can select the transfer functions for the F, P and H blocks (see Figures 1.7, 1.8) in the control loop; we will call it "**system import window**";
- one for importing a predefined/designed controller, in ZPK format, named "**controller import window**";
- and one for setting up a design spec, called "**constraint window**".

1.3.1.Main window

The main window is used to set up the design tool and it is shown in Figure 1.6. As you see, the window contains three menus (File, Tools and Help), zooming tool, axis for plotting the open loop Nichols curve, button for loop configuration, system definition frame, controller gain frame, Ngrid activation/deactivation checkbox, compensator editor activation button, responses frame, constraints frame, and status frame .
In the following subchapters we will discuss the features available in this window .

1.3.1.1. FILE menu

This menu is used for file operations and it has the following options: import, export of processes and controllers, Nichols plot/main window prints and, of course, application exit .
The "**Import**" option of this menu can be called to get transfer functions from the Matlab workspace. It has two submenus: one for importing a process and another one for importing a controller, in both cases placed in the blocks from the control loop shown in Figure 1.7 or Figure 1.8. In case that system import is requested the **system import window** will be activated and in case of controller import request, the **controller import window** will be shown (see Figures 1.12 and 1.13).

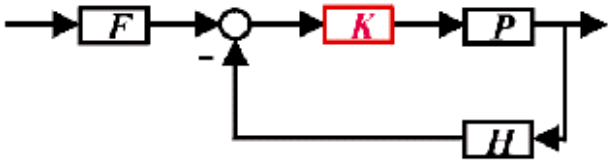

Figure 1.7 Control loop configuration

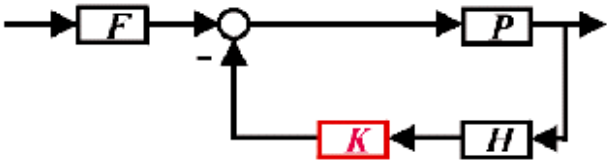

Figure 1.8 Control loop configuration

The "**Export**" option of the File menu allows user to save current loop configuration by saving F, P and H transfer functions in Matlab workspace, in case that system export is requested or by saving K transfer function, in case of controller export request. These transfer functions can be found in the Matlab workspace with the following names: sysF, sysP, sysH and sysK .

"**Print Nichols**" and "**Print**" options of the File menu will open the Matlab print preview window. First option will print only the Nichols curve, while the second one will print the **main window** as it is displayed .

"**Exit**" ends the application and closes all sub windows of FRTool .

1.3.1.2. Tools menu

The Tools menu has two submenus: the “Constraints” submenu that allows user to define the design specs (robustness, overshoot, settling time, phase margin and gain margin) and the “Simulation” submenu that allows user to define the final time for closed loop simulations. FRTool will automatically determine a final time for simulation but in case this value is not satisfactory, the user can define his own final time. For defining these restrictions FRTool will display a **constraint window** as shown in Figure 1.11 .

1.3.1.3. Help menu

At this time, only general help and release information are available via the options of this menu but home page navigation and other features will be added in a future version.

1.3.1.4. Nichols plot axes

Once the system and/or the controller are defined, the Nichols curve of the open loop system is displayed in the **main window**, to guide the user in the design process. The design specs (robustness, settling time, overshoot, gain margin and phase margin) will be also displayed as graphical restrictions on the same plot (see Figure 1.6). Watching this plot the designer can see the effect of changing controller parameters and can adjust these parameters until he fulfils all design specs .

1.3.1.5. Loop configuration button

The button displays continuously the current loop configuration (see Figures 1.7 and 1.8) and allows user to change the position of the controller from direct loop to feedback. Placing the process in different blocks on the control loop and changing the loop configuration, the user can obtain closed loop responses for setpoint and perturbances in different points of the control loop.

1.3.1.6. System definition frame

The system contained in the P block of the control loop can be defined as well using the system definition frame. In this case, only the numerator and denominator of the P system will be affected and no dead-time systems can be defined. Once a system is imported using the menu option (see import feature), these two textboxes will be

disabled and they will be re-enabled only when all the systems placed in F, P or H blocks are erased.

1.3.1.7. Controller gain frame

This frame contains a textbox with the value of the gain, two buttons which allow increasing and decreasing of the gain and a textbox for the step size (how much will be added or subtracted to/from the current value of the gain, to obtain the new value). Another way to change the controller's gain is dragging the bandwidth frequency with the mouse (only vertical movement is possible because this is the effect of changing the gain of the open loop).

1.3.1.8. Ngrid and controller editor activation frame

Clicking on the Ngrid checkbox the user can show or hide the constant gain and phase curves for the Nichols plot (the so-called Ngrid).
The "Edit compensator" button can be used to open or reopen the **design window**.

1.3.1.9. Responses frame

Using the checkboxes available in this frame, the user can select which responses or characteristics have to be displayed. For instance, if step response is requested, the user will click on the "Step" checkbox to activate this type of response. Multiple responses and/or characteristics can be active at the same time and they will be displayed in the same window: the "**responses window"**.

1.3.1.10. Constraints frame

This frame is similar with the "responses frame" but it is used to notify FRtool that a certain design spec should be taken into account when drawing the graphical constraints. If the value of that design spec is not already set, the **constraint window** will be displayed to input the value of that design spec. Even if defined, a design spec can be omitted if the corresponding checkbox is unchecked.

1.3.1.11. Status frame

This frame will be used by FRtool to communicate with the user. For example, if a gain equal to zero will be set, FRtool will display a message to inform the user that he should change the gain to another value because that one is not allowed.

1.3.2.Responses window

Simulations of closed loop responses and characteristics can be seen in the "**responses window"** (see Figure 1.9). This window is shown/hidden automatically by FRtool as follows: if no responses or characteristics are needed, the window will be hidden; once at least one response is requested, it will be automatically shown.
All requested responses and characteristics will share the same window and will be updated every time a change is made to the system or controller parameters.
Right clicking on a characteristic (plot), users can select features like grid on/off, peak

response, settling time, rising time or zoom.
The **Print** option in the **File menu** of the "**responses window**" will display the Matlab print preview window to print all the responses currently displayed.

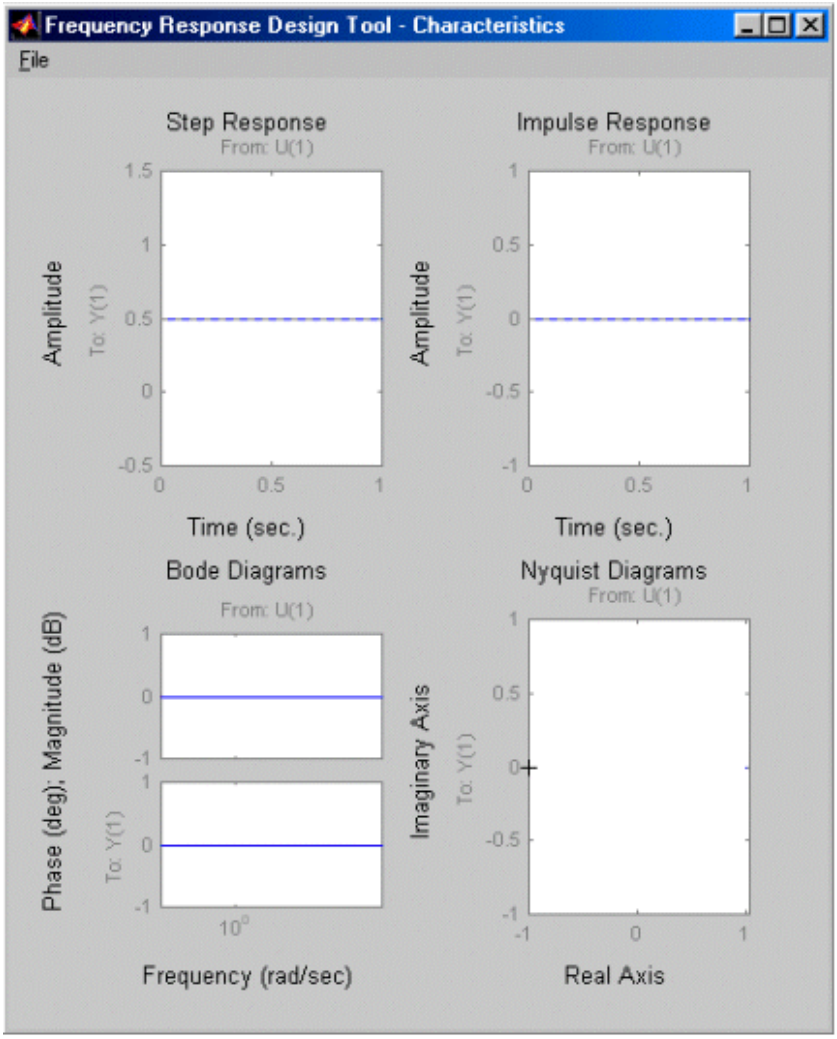

Figure 1.9. Responses window of FRTool

1.3.3. Design window

This is the second most important window of the frequency design tool (see Figure 1.10) because it allows users to change the position of the controller poles and zeros and also to add/remove new/existing poles and zeros. The limits of the axes will be adjusted in correlation with the pole/zero placements and current limits will be changed using the zooming tool available in this window, which works in the same way as the one available in the **main window**.
Using drag & drop, the user can change the position of the poles and zeros and while dragging the roots he can see the changes of the Nichols curve in real time. FRtool will take care of complex conjugated poles, moving the conjugated pole in the appropriate position. By releasing the mouse button the new position of the pole or zero will be set and the responses window will be updated, too.

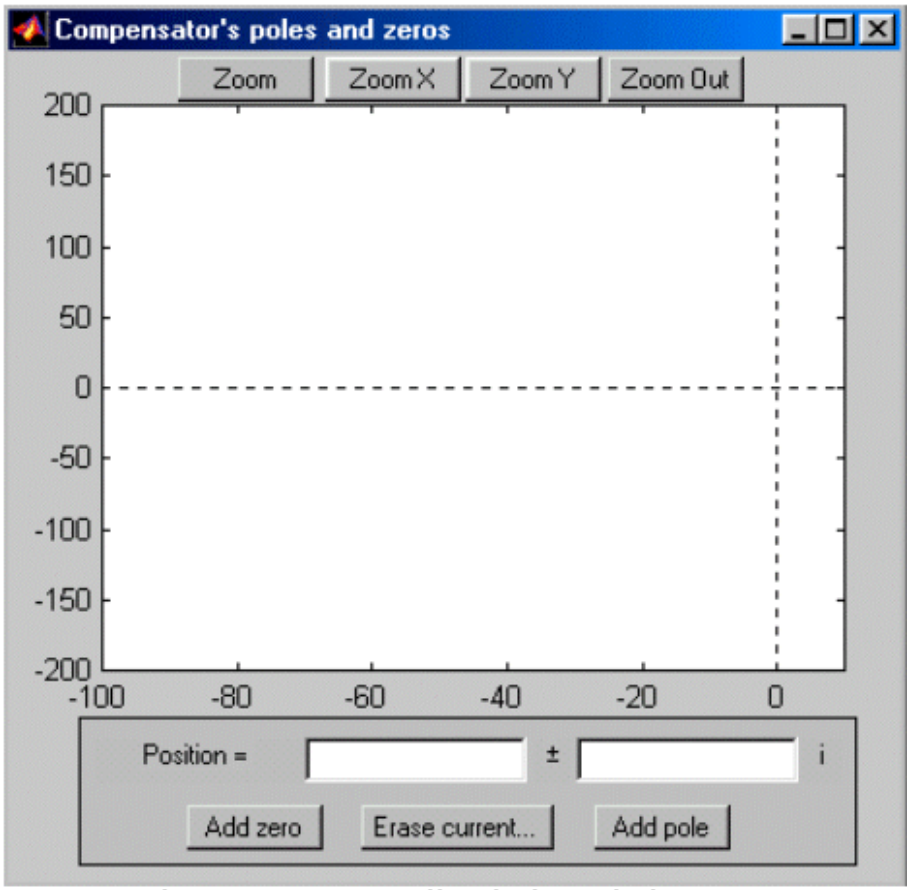

Figure 1.10. Controller design window

1.3.4.Other windows of FRTool

Whenever a design specification has to be defined FRTool displays a window similar to the one shown in Figure 1.11. The window will contain the restriction's name, type (<, >, =, <=, >=), value and measurement units. Figure 1.11 shows the overshoot case and it can be read as "Overshoot has to be lower than ___ percents" .

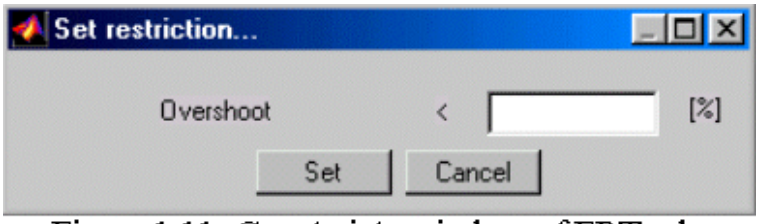

Figure 1.11. Constraints window of FRTool

As we mentioned before, the process can be placed in three positions P, F, H on the control loop (see Figure 1.7 or 1.8). Therefore, the **process import window** shown in Figure 1.12 has at its left side a list of processes defined as "TF" or "ZPK" in the Matlab workspace and a set of three buttons, one for each block from the control loop. To insert a process from the list into block P, it has to be selected with the mouse and the " à P" button has to be pressed. When everything is set, the "Use" button will validate the loop configuration.

Similar with the **process import window** is the **controller import window**. The difference is that, in this case, the list will contain only “ZPK” systems defined in the workspace. To use a predefined controller, it has to be selected form the list and inserted into K block with “ à K”.

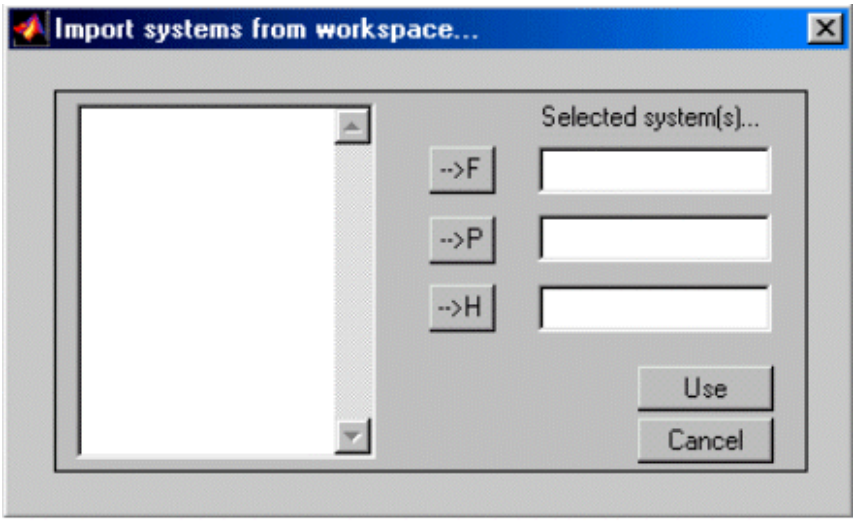

Figure 1.12. System import window

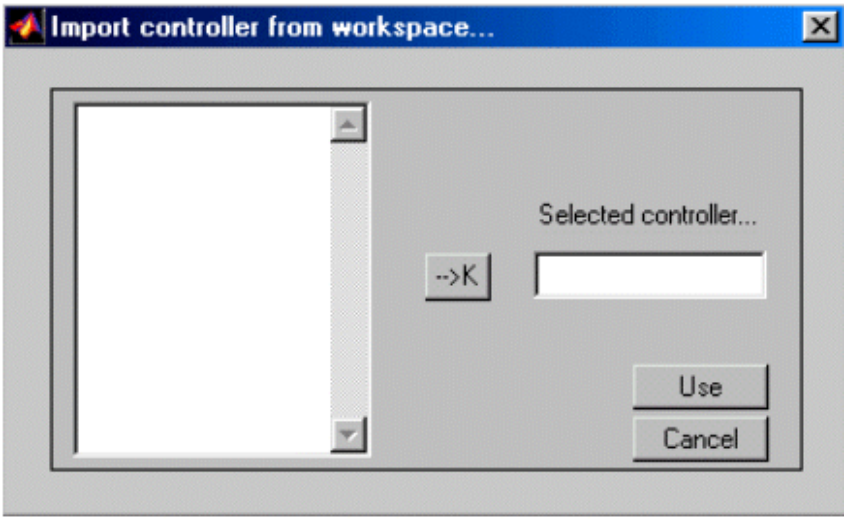

Figure 1.13. Controller import window

1.4. Design example (application)

In most of the cases the easiest way to learn how to use software is to follow a short demonstration. Therefore, we picked an application and we will try to give a demo of the design procedure.
Task: considering that the plant has the transfer function given in relation 1.9, design a phase-lead controller such that the unity feedback closed loop system will satisfy the following specifications:

- maximum overshoot < 5 %
- settling time < 0.8 seconds

$$G(s) = \frac{2500}{s(s+25)} * e^{-0.2S} \qquad (1.9)$$

Supposing that FRtool is already installed and working properly on our system we define the process in the Matlab workspace using relation 1.10 .

$$Sys = tf(2500,[1\,25\,0], inputDelay, 0.2), \qquad (1.10)$$

If FRtool is not started yet, it can be started with “frtool” command. Once the tool’s main window is visible we can start using the tool by importing the transfer function of the process. Clicking on “File/Import/System” menu the **system import window** will appear with the “Sys” transfer function in the list. After clicking on the “Sys” name, on the “ à P” button and, finally, on the “Use” button the system is placed into the P block of the control loop and in the main window is displayed the Nichols plot of the open loop. At this time the open loop contains only the P transfer function (our system).
Next step is to define the design specs. Clicking on the “Ts” checkbox, **constraint window** will be displayed to enter the value of the settling time spec, because it was not defined yet. After entering this value we proceed with the overshoot design spec in the same way. Afterwards, the **main window** of the tool will be the one shown in Figure 1.14.
From now on we have to pick the structure of the controller and design its parameters. We have to design a phase-lead controller defined by relation 1.11 which leads us to relation 1.12 .

$$K(s) = \frac{1+Ts}{1+\alpha Ts}, \quad \alpha < 1 \quad (1.11)$$

$$|Pole| > |Zero| \qquad (1.12)$$

To have the structure defined in relation 1.11 the controller’s pole and zero have to be both pure real roots. Furthermore, to fulfill relation 1.12 we will choose the pole’s location (-25) and the zero’s location (-2). To define them we have to go to the **controller design window** and do the following actions: write –25 in the real part textbox and then press “add pole” button, to define the pole and write –2 in the same textbox and press “Add zero” button to define the zero. At this time the **controller design window** will look like in Figure 1.15 and the **main window** will be similar to Figure 1.16.

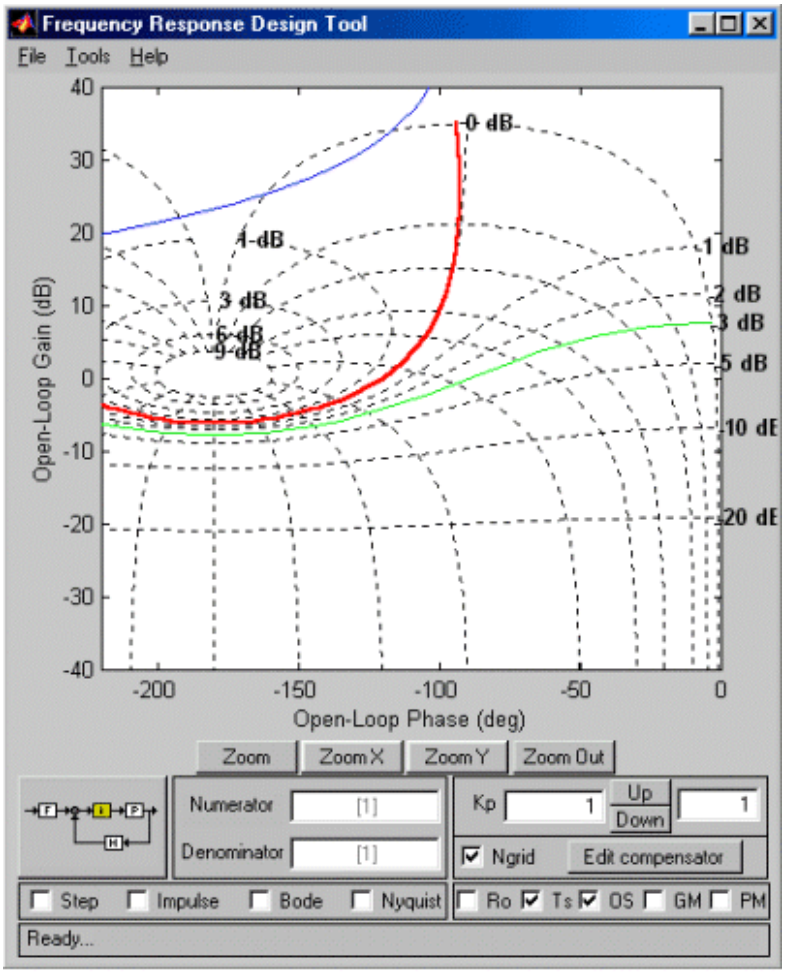

Figure 1.14. Main window of FRTool

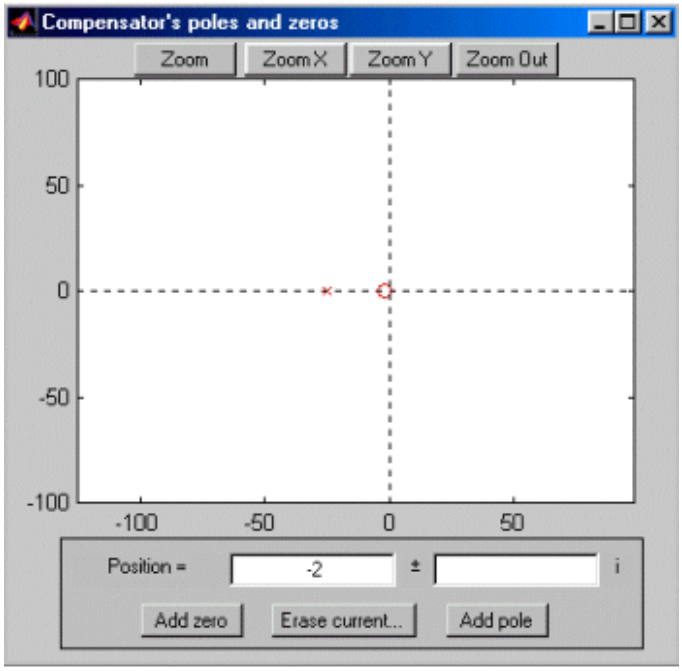

Figure 1.15. Controller pole/zero placement

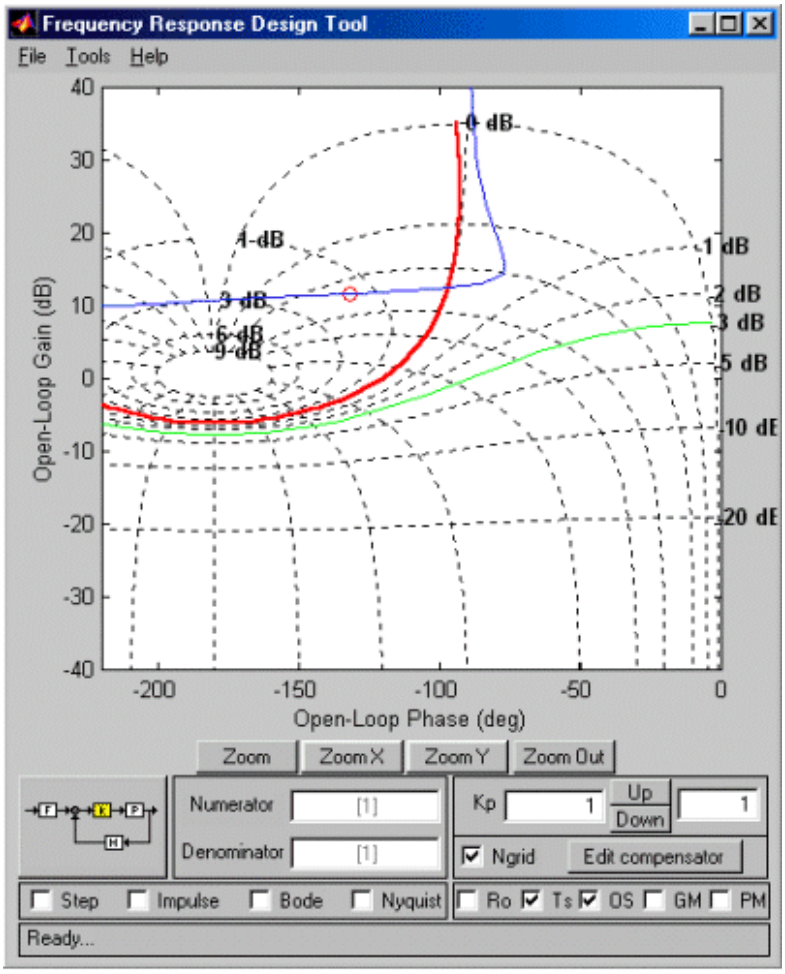

Figure 1.16. Nichols curve of the K*P open loop

We don't know yet the effect of changing the controller's zero, pole and gain (note: the static gain of the phase-lead compensator has not been shown explicitly in formula 1.11). Now we shall test the effect of each one. First we will move the zero from (–2) to (–5) and we shall try to see the effect on the Nichols curve. Figure 1.17 shows the new position of the Nichols curve.
Figure 1.18 shows the effect of placing the zero in (-1). We can see that the Nichols shape becomes more convex when he zero becomes more negative and that the Nichols curve starts having a peak when the zero is less negative .

Next step is to see the effect of the pole's position on the Nichols curve. Therefore, we will first move the pole to (-60) and will obtain the Figure 1.19. The effect of placing the pole at (-10) is shown in Figure 1.20.
At this time we know the effect of changing the zero and pole's position and we also know from theory the effect of the loop gain on the Nichols plot (illustrated in Figure 1.21 and 1.22): when increasing the gain (higher values than 1) the curve will be shifted up, when decreasing the gain (values between 0 and 1) the Nichols curve will be shifted down.

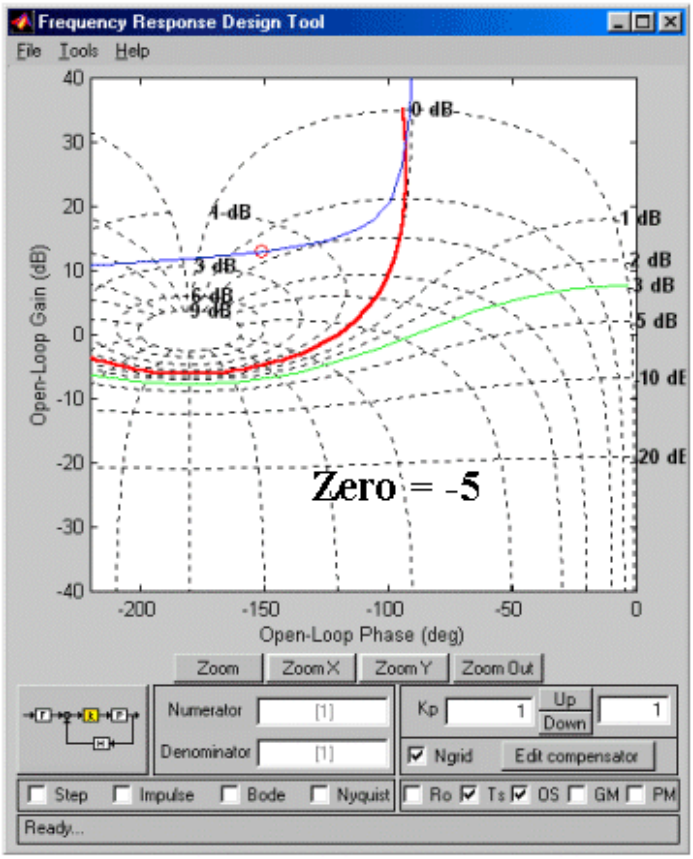

Figure 1.17. New Nichols curve with zero changed to -5

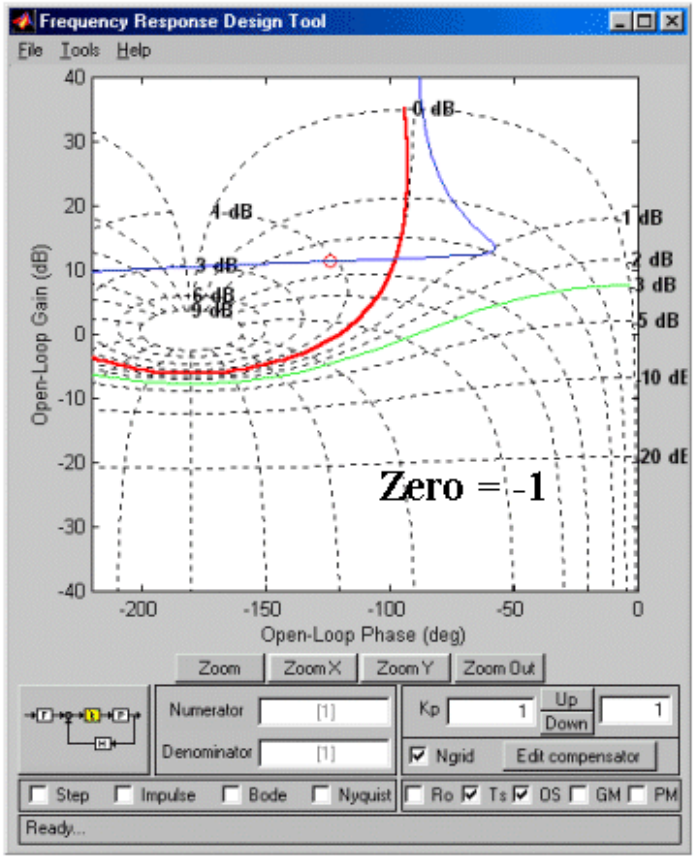

Figure 1.18. Nichols curve when zero is placed in (-1)

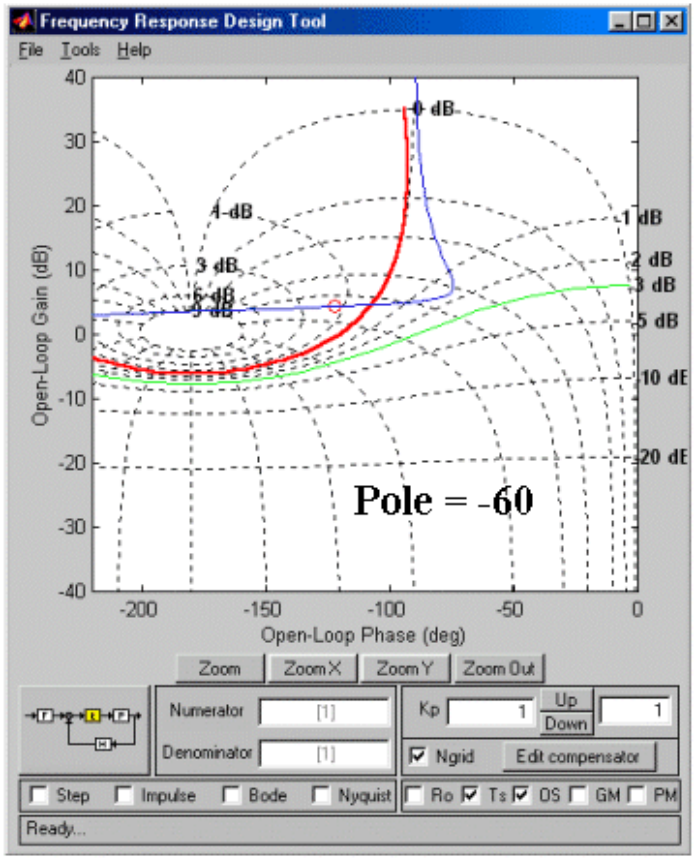

Figure 1.19. Nichols curve when pole is placed at -60

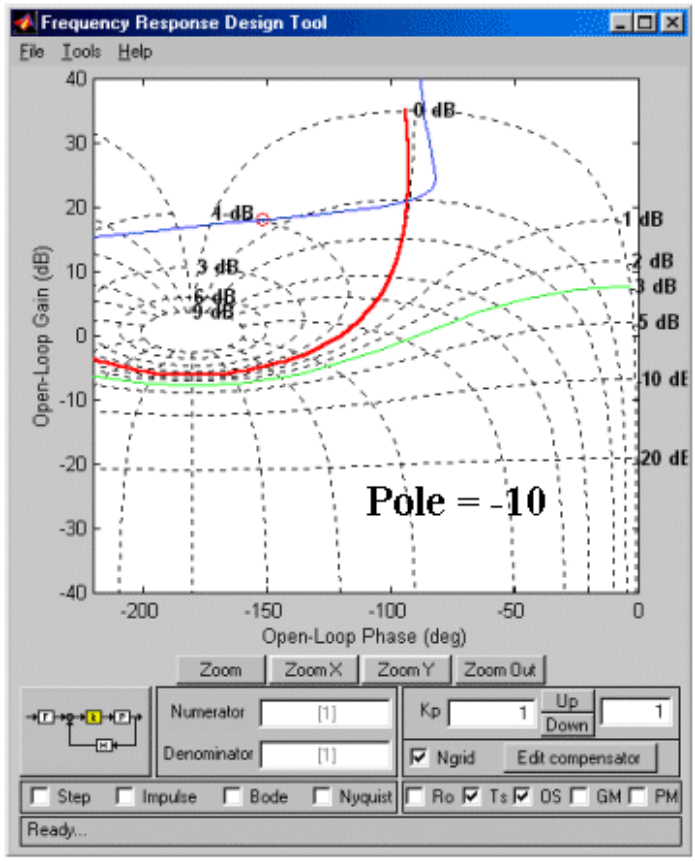

Figure 1.20. Nichols curve with controller pole at -10

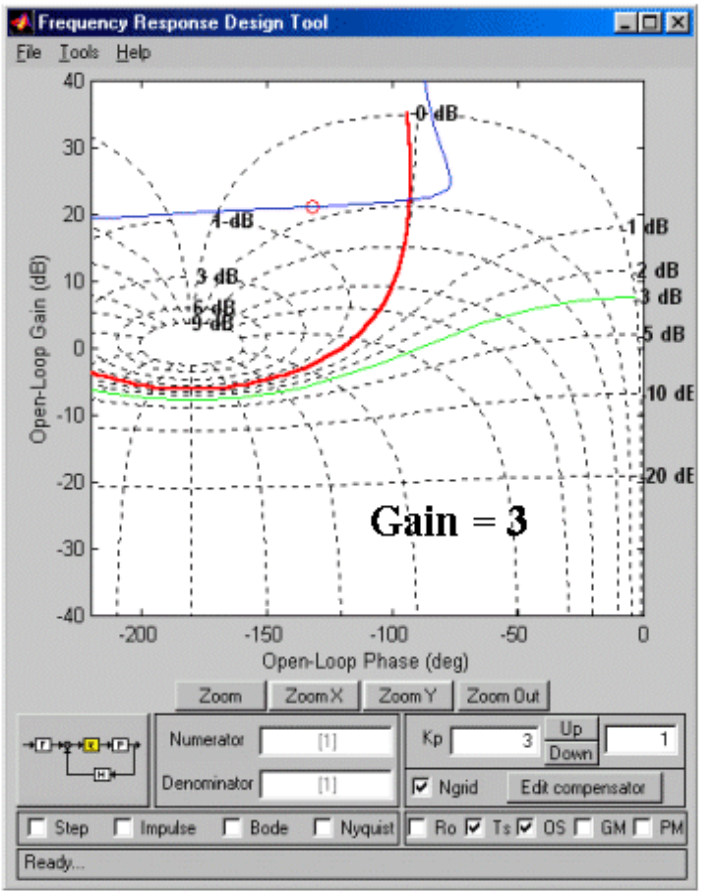

Figure 1.21. Nichols curve shifted with a gain of 3

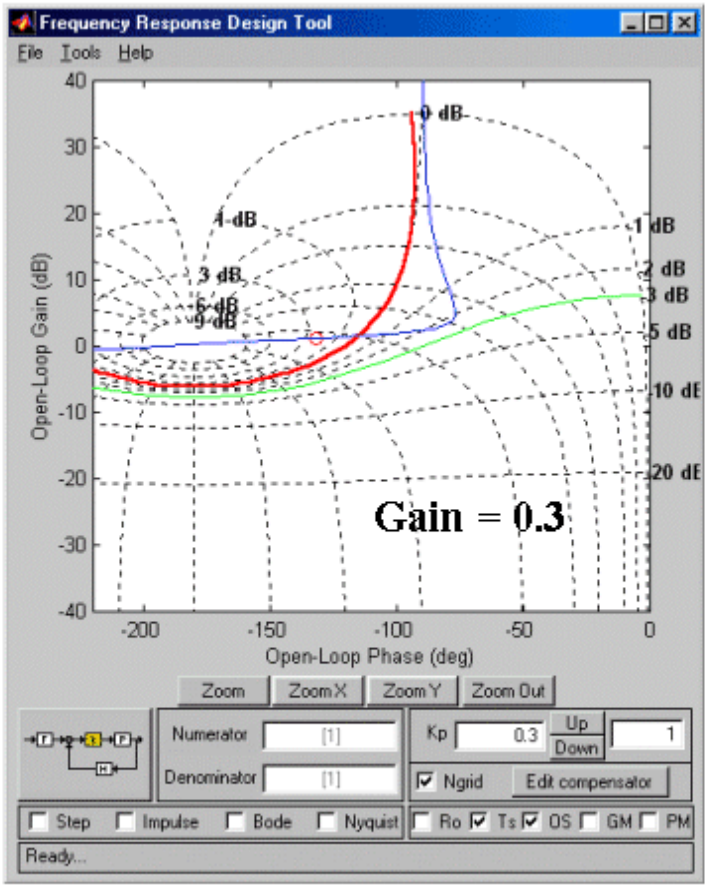

Figure 1.22. Nichols curve shifted with a gain of 0.3

Playing a few minutes with the pole/zero placement and gain we can shape the Nichols curve so that all design specs are fulfilled. The picture of the resulting Nichols curve is shown in Figure 1.23. You can see that the curve does not cross the OS restriction and that the bandwidth frequency is above the –3dB green line. This means that all design

specs are fulfilled and we can prove this by taking a look at the closed loop response of the system (see Figure 1.24). The controller parameters resulting from this design exercise are listed in Figure 1.23 and the values of overshoot and settling time obtained are listed in Figure 1.24 .

1.5. FAQ

1.5.1.How can I tune controller parameters?

Controller parameters are gain, poles and zeros. The gain can be tuned using the controller gain frame placed in the main window or by dragging the red circle (bandwidth frequency) displayed on the Nichols curve .
The controller poles and zeros can be easily tuned using drag & drop features available in the **controller design window**.

1.5.2.How can I define restrictions (design specs)?

When selecting a restriction, FRtool checks first if there is a value defined for this restriction. If no value is found, the "**constraint window**" is displayed, to force the user to give a value to the restriction .
Another way is to select the “Tools/Specs” submenu options and insert the value in the window that is displayed. In this case the checkbox that activates the restriction has to be clicked, otherwise the spec is not displayed .

1.5.3.How can I erase all systems defined in the F, P, and H blocks?

By simply opening the **system import window** from “File/Import/System” menu option and clicking on **Use**, without selecting any system, the blocks from the control loop will be set to empty .

1.5.4.How do I have to define the system in workspace?

In order to be able to import a system, the user has to define it as TF or ZPK. Only these types of systems are displayed in the **system import window**'s list .

1.5.5.How does the controller have to be defined?

In order to be able to import a controller, the user has to define it as ZPK. Only this type of system is displayed in the **controller import window**'s list .

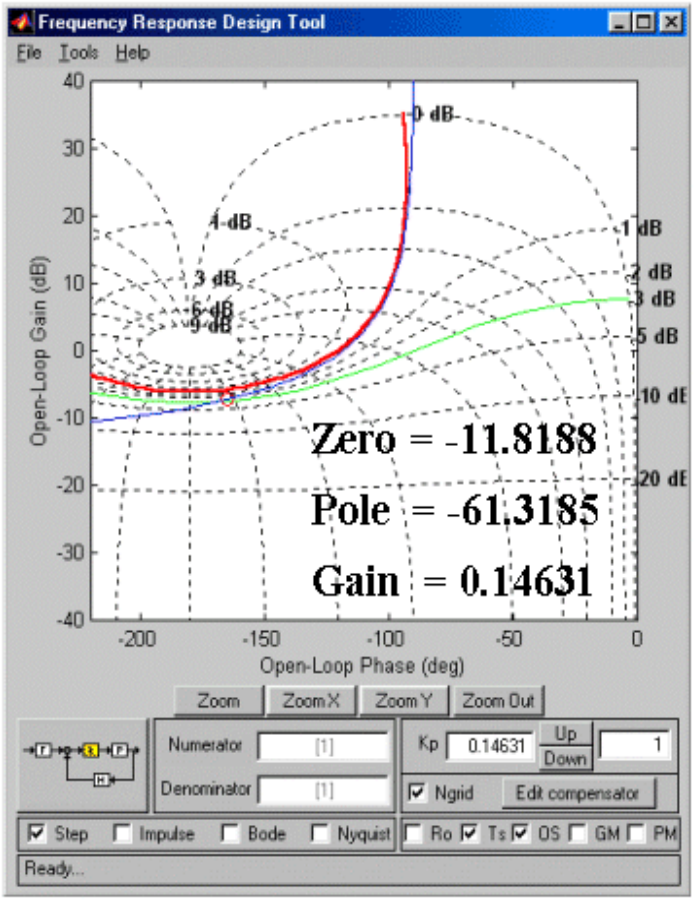

Figure 1.23. FRTool's main window with all specs fulfilled

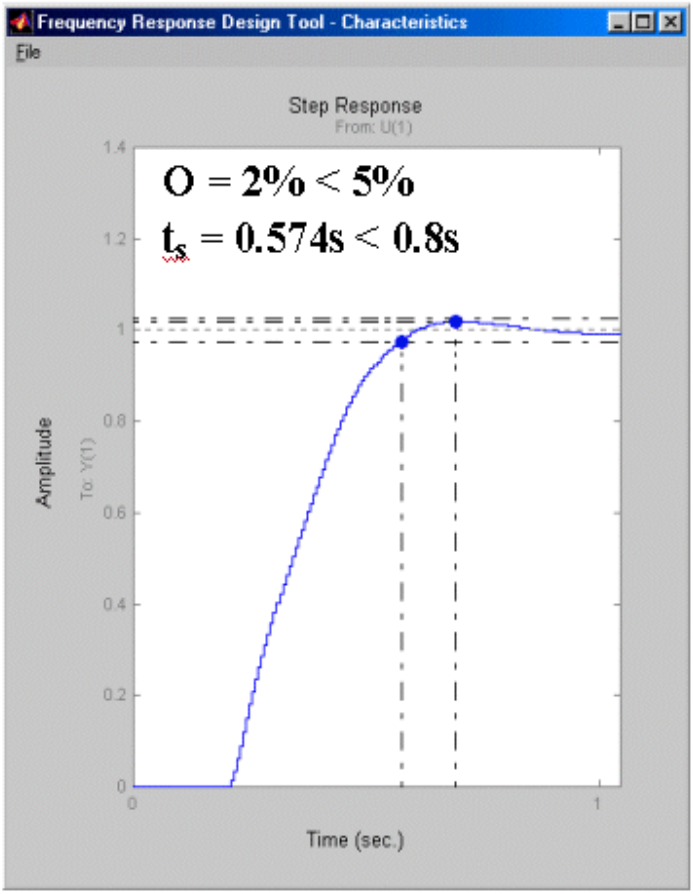

Figure 1.24. FRTool's responses window - Step response

1.5.6.How will I find my exported systems and controller?

On "File/Export/System" request, FRtool will send to Matlab workspace the systems from all blocks except the K block, and the given names will be: "**sysF**", "**sysP**" and "**sysH**".
On "File/Export/Controller" request, FRTool will send to the same Matlab workspace the controller in ZPK format with the "**sysK**" name .

1.5.7.Why, after I define the settling time, it is not displayed on the Nichols curve?

Settling time restriction's graphical equivalent is considered to be the bandwidth frequency, which is computed using damping factor and natural frequency. To compute the damping factor you have to define a maximum overshoot. Therefore, you cannot use only the settling time restriction, without the overshoot restriction.
However, overshoot restriction can be used without defining settling time spec.